Dispersion Analysis of the Linear Vane-Type Waveguide Using the Generalized Scattering Matrix

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***Abstract*— The dispersion characteristics for the linear vane-type waveguide are determined using the generalized scattering matrix (GSM) formed with the mode matching algorithm. This dispersion analysis technique includes determining the eigenvalues (cutoff frequencies) for the various waveguide modes that can propagate on the circuit, as well as forming a determinantal equation for a single period of the circuit from which the system normal mode dispersion characteristics are determined. The resulting GSM is easily manipulated for determining eigenvalues for single or multiple periods of a periodic circuit using either a perfect electrical conductor (PEC) or a Re-entrant boundary condition. This boundary condition formulation using the GSM provides a generalized eigenvalue technique for 2-D and 3-D structures. Similarly, the GSM is easily manipulated to yield a new analytic expression for a determinantal equation to predict the dispersion of the system normal modes for the periodic circuit. The accuracy of the GSM eigenvalue and dispersion solution techniques are limited by the frequency resolution of the simulation and the relative convergence (RC) criterion.**

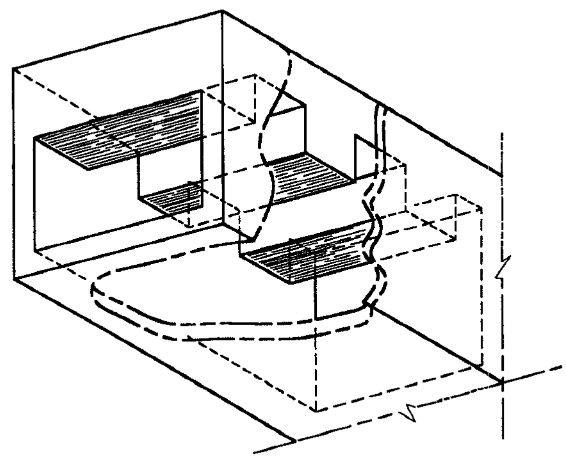

Fig. 1. 3-D linear vane-type waveguide geometry.

I. Introduction

The DISPERSION characteristics of a periodic circuit, such as the vane-type waveguide, may be determined with a variety of experimental, analytic, and numerical techniques. Experimental methods [1] employing the resonance technique are commonly used to measure the dispersion characteristics for a periodic circuit after construction. Dispersion measurements are necessary for comparing the design objectives with those obtained from the physical hardware. The correlation between the dispersion design objectives and that obtained from the physical hardware is strongly dependent upon the design technique used for the periodic structure. Accurate design techniques are required to maximize the correlation between the dispersion design objectives and the experimentally measured dispersion information.

Historically, the vane-type waveguide has been designed using a variety of techniques. The selected design technique depends upon the configuration of the vane-type waveguide, which may exist in either a closed end-space or in an open end-space configuration. Similarly, the vane-type waveguide may exist in a cylindrical re-entrant format such as that used with the magnetron oscillator [2], or in a linear format for amplifier [3], [4], antenna [5], and filter [6] applications. This analysis deals with the linear closed end-space vane-type waveguide configuration shown in Fig. 1, which is fundamentally a rectangular waveguide with a corrugated surface capable of propagating slow waves.

This configuration can be analyzed to determine the circuit dispersion characteristics using variational techniques yielding equivalent circuit models [2], [7] and field matching models [2], [4], [7], [8] from which the dispersion characteristics of the circuit are determined. These modeling techniques are tailored to determine the dispersion of selected modes defined by the basis functions used to represent the modes propagating on the circuit. The assumptions used with these modeling techniques typically provide at best a 1% error between the predicted and measured dispersion characteristics of the circuit. Expensive experimental techniques are required to resolve the resulting design errors. Problems are also often encountered in the laboratory with the revelation that competing cavity and waveguide modes exist in the passband of the desired system normal mode, causing interference and power loss from the desired mode of operation for the circuit. These problems are typically not identified using variational approaches for designing vane-type waveguides.

These dispersion design problems can be resolved using generalized numerical electromagnetics modeling codes involving finite difference or finite element algorithms. Recently published information [9], [10] concerned with using these algorithms as design tools to determine the dispersion characteristics of slow wave modes for periodic circuits illustrates the ability to model arbitrarily shaped structures with the codes, but also discusses the errors typically obtained with these algorithms. The 3-D finite difference MAFIA code [9] has been recently used to model the dispersion characteristics of

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a coupled-cavity traveling wave tube slow wave circuit. The MAFIA dispersion algorithm was able to obtain an error range between measured and calculated dispersion information of 0.0% to 1.6% over the passband for the slow wave circuit mode. Similarly, the 2-D finite difference QUAP and 3-D finite difference ARGUS codes [10] were able to model a hole-and-slot waveguide coupled crossed-field amplifier slow wave circuit. These finite difference dispersion algorithms were able to obtain an error range between measured and calculated dispersion information for the slow wave mode of 0.1% to 2.1%. These errors are related to discretizing the periodic circuit structure space to properly represent the problem boundaries to the generalized dispersion algorithms. Errors of this type may be minimized by increasing the mesh density for the finite difference simulation [10], or by using alternative periodic circuit dispersion algorithms such as the GSM mode matching algorithm.

Many dispersion design problems can be resolved using the mode matching algorithm [11], which results in the formation of the generalized scattering matrix (GSM) [11]–[14] for a periodic circuit. The vane-type waveguide to be analyzed is considered as a two port circuit in this study. The resulting GSM for this two port periodic circuit yields an analytic expression for the system eigenvalues using either a perfect electrical conductor (PEC) or a Re-entrant boundary condition. Similarly, the GSM yields a new analytic expression for the determinantal equation of the periodic circuit, from which the dispersion of the system normal mode or modes is determined. The accuracy of the eigenvalues and determinantal equation determined from manipulation of the GSM is a function of the frequency resolution for the simulation and the relative convergence (RC) criterion [7], [11].

Section II presents the details of how the GSM is manipulated to calculate eigenvalues using either a PEC, or a Re-entrant boundary condition. Section III presents the details of how the GSM is manipulated to yield a determinantal equation from which the dispersion characteristics of the system normal modes of a periodic circuit are determined. Section IV provides a demonstration of the eigenvalue and dispersion algorithms using the GSM in comparison to measured dispersion information for a linear closed end-space vane-type waveguide. Finally, Section V summarizes this work and illustrates the generality of the analytic technique for analyzing the dispersion characteristics of periodic circuits using the mode matching algorithm when a GSM is determined.

II. Eigenvalue Analysis Using the GSM

A. Introduction to the GSM for Eigenvalue Analysis

The mode matching algorithm employing the GSM is typically used to determine the normal mode scattering characteristics of two or more cascaded waveguides. Each region or section of waveguide forming the circuit typically contains a different number of modes in the normal mode expansion of the electromagnetic fields. The number of modes used for each region is determined from the RC criterion. A typical circuit configuration for this analytical technique is shown in

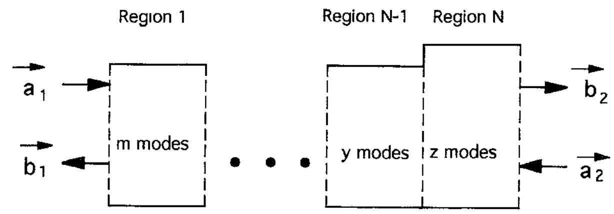

Fig. 2. Cascaded waveguide configuration for mode matching analysis.

Fig. 2, where each region of the circuit is composed of a unique normal mode expansion for the type of waveguide (i.e., rectangular, cylindrical, coaxial, elliptical, parallel plate, etc.), and each region may contain a different number of modes to properly satisfy the RC criterion.

The resulting GSM for the geometry shown in Fig. 2 is represented as

$$\begin{bmatrix} \overline{\overline{S_{11}}} & \overline{\overline{S_{12}}} \\ \overline{\overline{S_{21}}} & \overline{\overline{S_{22}}} \end{bmatrix} \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \end{bmatrix} = \begin{bmatrix} \vec{b}_1 \\ \vec{b}_2 \end{bmatrix}. \tag{1}$$

This matrix form for the cascaded waveguide configuration shown in Fig. 2 represents only the scattering parameters ($\overline{\overline{S_{11}}}$, $\overline{\overline{S_{12}}}$, $\overline{\overline{S_{21}}}$, $\overline{\overline{S_{22}}}$) and scattering variable ($\vec{a}_1$, $\vec{a}_2$, $\vec{b}_1$, $\vec{b}_2$) amplitudes for the first (Region 1) and last (Region N) regions of the circuit. The GSM formulation of the mode matching algorithm involves matrix manipulation techniques that permit the scattering matrices for the uniform lengths of waveguide and waveguide junctions to be combined in a compact, unconditionally stable matrix representing the two port scattering characteristics for the circuit being studied.

B. GSM Eigenvalue Problems Using the PEC Boundary Condition

The GSM and corresponding scattering variable vector representation shown in (1) is for each mode contained in the first and last regions of a series of cascaded waveguides forming a circuit such as that shown in Fig. 2. This matrix representation typically assumes that the waves leaving the simulation are terminated in a perfectly matched load.

This load condition can be manipulated with the GSM algorithm to represent this as an eigenvalue problem. If the first and last region ports are replaced by PEC's creating a perfectly reflecting boundary condition for each mode in these two regions, then the vector scattering amplitudes are defined at the ports of the circuit as

$$\vec{b}_1 = -\vec{a}_1 \tag{2}$$

$$\vec{b}_2 = -\vec{a}_2. \tag{3}$$

These scattering vector identities are substituted into (1) yielding

$$\begin{bmatrix} \overline{\overline{S_{11}}} & \overline{\overline{S_{12}}} \\ \overline{\overline{S_{21}}} & \overline{\overline{S_{22}}} \end{bmatrix} \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \end{bmatrix} = \begin{bmatrix} \vec{b}_1 \\ \vec{b}_2 \end{bmatrix} = \begin{bmatrix} -\vec{a}_1 \\ -\vec{a}_2 \end{bmatrix}. \tag{4}$$

This equation is of the form suitable for determining the eigenvalues for a geometry terminated with a PEC boundary condition. In this procedure, the frequency is swept in the real or complex plane to determine the eigenvalues for the problem geometry. This is accomplished by taking the determinant of (5), where as the frequency is swept, eigenvalues are found as the determinant goes to zero

$$\begin{vmatrix} \overline{\overline{S_{11}}} + \overline{\overline{1}} & \overline{\overline{S_{12}}} \\ \overline{\overline{S_{21}}} & \overline{\overline{S_{22}}} + \overline{\overline{1}} \end{vmatrix} = 0. \tag{5}$$

The GSM eigenvalue equation shown in (5) is used to determine the normal mode cutoff frequencies (eigenvalues) for any structure modeled with the mode matching algorithm, assuming that the structure is terminated with PEC boundaries. Using this algorithm in conjunction with the GSM determined for a single period of a periodic structure will permit the identification of the cutoff frequencies for the various modes capable of propagating on the structure. In particular, the band-pass dispersion characteristic commonly encountered with a surface wave mode will permit the determination of the cutoff frequency for the 0-mode and for the π-mode. Determination of these two frequencies defines the passband for the surface wave mode of the circuit being analyzed.

C. GSM Eigenvalue Problems Using the Re-entrant Boundary Condition

The Re-entrant boundary condition is analyzed in a similar manner to that of the PEC boundary condition. However, the Re-entrant boundary condition requires that any wave leaving Port 1 of a two port circuit re-enter the simulation at Port 2 with no respective amplitude or phase change. The same phenomenon is true for the inverse problem of a wave exiting Port 2 and re-entering Port 1. This problem requires symmetry of the boundary conditions for the first and last regions, where the two regions must be defined as physically identical waveguide regions with the same normal mode content.

This boundary condition requires that the vector scattering amplitudes be represented as

$$\vec{a}_1 = \vec{b}_2 \tag{6}$$

$$\vec{a}_2 = \vec{b}_1. \tag{7}$$

These two vector scattering amplitude identities are applied to the GSM yielding

$$\begin{bmatrix} \overline{\overline{S_{11}}} & \overline{\overline{S_{12}}} \\ \overline{\overline{S_{21}}} & \overline{\overline{S_{22}}} \end{bmatrix} \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \end{bmatrix} = \begin{bmatrix} \vec{b}_1 \\ \vec{b}_2 \end{bmatrix} = \begin{bmatrix} \vec{a}_2 \\ \vec{a}_1 \end{bmatrix}. \tag{8}$$

The symmetry conditions defined for the Re-entrant boundary condition force the transmission and return loss sub-matrices of the GSM to be square, and manipulation of the transmission terms is now possible. This problem is of a form suitable for determining the eigenvalues for a re-entrant geometry. As before, the frequency is swept in the real or complex plane to determine the eigenvalues

$$\begin{vmatrix} \overline{\overline{S_{11}}} & \overline{\overline{S_{12}}} - \overline{\overline{1}} \\ \overline{\overline{S_{21}}} - \overline{\overline{1}} & \overline{\overline{S_{22}}} \end{vmatrix} = 0. \tag{9}$$

The Re-entrant boundary condition used with the GSM for determining eigenvalues is generally not applicable for analyzing a single period of a periodic circuit. The two surface wave resonances being sought for the purposes of this analysis are the 0-mode and the π-mode. The Re-entrant boundary condition requires that the signal magnitude and phase be equivalent at each port of a two port circuit. Therefore, the eigenvalue for the 0-mode is determined with this algorithm because the signal magnitude and phase are equivalent at each port. However, the π-mode is not found with the Re-entrant boundary condition, because the resulting electromagnetic field magnitude is equivalent at each port, but exhibits a 180° phase shift between periods. Therefore, the PEC boundary condition for the GSM provides a more generalized and, hence, preferable eigenvalue formalism for analyzing a single period of a periodic circuit.

III. Dispersion Analysis of a Periodic Circuit Using the GSM

A. Classical Dispersion Analysis Using the Transmission Matrix

The determinantal equation for a periodic circuit is typically derived from the transition (transfer) matrix for a single period of a periodic circuit. The transition or transfer matrix is equivalent in form to the transmission matrix for a circuit, as addressed by Collin [6] for voltage-current and wave amplitude matrix relations. The wave amplitude transmission matrix relationship has been adapted to the definition of scattering matrix variables for the circuit illustrated in Fig. 2. The GSM for the circuit shown in Fig. 2 is represented as previously shown in (1).

The transmission matrix representation relates the Port 1 scattering (wave amplitude) variables to the Port 2 scattering (wave amplitude) variables as

$$\begin{bmatrix} \overline{\overline{T_{11}}} & \overline{\overline{T_{12}}} \\ \overline{\overline{T_{21}}} & \overline{\overline{T_{22}}} \end{bmatrix} \begin{bmatrix} \vec{a}_1 \\ \vec{b}_1 \end{bmatrix} = \begin{bmatrix} \vec{b}_2 \\ \vec{a}_2 \end{bmatrix}. \tag{10}$$

The general form of the transmission matrix in (10) relates the input port (Port 1) forward and reflected mode amplitude variables to the output port (Port 2) forward and reflected wave amplitudes. The matrix form shown in (10) will be referred to as the generalized transmission matrix (GTM), since it is in a form similar to that of the GSM represented in (1).

The GTM has been extensively evaluated in comparison to the GSM for the mode matching algorithm. This problem has been addressed by Mansour [15] for the general analysis of circuits involving the mode matching algorithm. The transmission matrix has many advantages over the scattering matrix, in particular for the analysis of a series of cascaded waveguide discontinuities. If a scattering matrix is determined for each junction and region of a circuit using the mode matching algorithm, then the matrices are combined through defined matrix manipulations [12]–[14] to form the GSM. However, if a transmission matrix is determined for each region and junction, then the matrices are combined by simply multiplying the matrices [6], [15] in order of their occurrence for the circuit. This reduces the computational effort significantly and has demonstrated equivalent analytical results with the GSM [15] approach.

The cascaded waveguide scheme for the GTM is much simpler to implement than the GSM technique, but several problems exist with the GTM technique. Mansour [15] formulated the analytical technique with the capability to use an arbitrary number of modes in each region forming the GTM with the mode matching algorithm. Omar [16] first developed the GTM technique using an equal number of modes in each region of the circuit, but experienced problems satisfying the RC criterion. These two efforts have been recently reviewed by Dai [17], who confirmed that the effort by Omar [16] violates the RC criterion when an equal number of modes are used in each region for specific classes of problems. However, the ability to formulate the GTM with an unequal number of modes in each region of the circuit will not violate the RC criterion.

The formulation of the GTM by Mansour [15] and the classical formulation by Omar [16] suffer from a potential numerical instability due to the form of the GTM. The GTM is partially formed using transmission terms defining waveguide junctions, and uniform lengths of waveguide similarly defined for the GSM. The mode matching algorithm uses propagating as well as evanescent modes to define the electromagnetic fields at the plane of a discontinuity. The GTM contains exponential functions with positive arguments, which may result in numerical overflow for the evanescent modes in the matrix. This problem may lead to numerical instabilities with the GTM when evanescent modes are included in the mode matching process. This eventually led Mansour to abandon the GTM process.

Nevertheless, the GTM is commonly used to form a determinantal equation for a periodic circuit from which the system normal mode dispersion characteristics are determined. This is accomplished by selecting one port of the circuit as the defined reference plane for the dispersion analysis problem. The waves exiting this circuit and entering the next period are related by Floquet's theorem [1], [6], which relates the waves at the defined reference plane for the two periods by a complex phase constant. In particular, any wave leaving the first period of the circuit is related to the input wave amplitude for the second period of the circuit by a defined phase constant for the circuit length (L), and the two ports are related with respect to the reference plane as

$$\vec{b}_2 = \vec{a}_1 e^{-\Gamma_n L} \tag{11}$$

$$\vec{b}_1 = \vec{a}_2 e^{\Gamma_n L} \tag{12}$$

where $\Gamma_n = \alpha_n + j\beta_n$. Substitution of (11) and (12) into (10) with respect to the reference plane and simple matrix manipulation yields

$$\begin{bmatrix} \overline{\overline{T_{11}}} - \overline{\overline{e^{-\Gamma_n L}}} & \overline{\overline{T_{12}}} \\ \overline{\overline{T_{21}}} & \overline{\overline{T_{22}}} - \overline{\overline{e^{-\Gamma_n L}}} \end{bmatrix} \begin{bmatrix} \vec{a}_1 \\ \vec{b}_1 \end{bmatrix} = \begin{bmatrix} \vec{0} \\ \vec{0} \end{bmatrix} \tag{13}$$

where $\overline{\overline{e^{-\Gamma_n L}}}$ is a diagonal matrix defining the complex eigenvalues to be found expressing the dispersion of a periodic circuit.

This formulation of the eigenvalue problem is subject to numerical instabilities as previously described, but (13) demonstrates the form necessary in the mode matching formalism to solve for the dispersion characteristics of a periodic circuit. The GTM technique has been proven to be numerically unstable [7], [17], while the previous derivation of the GSM [12]–[14] illustrates that the GSM technique is unconditionally stable. This GSM matrix stability is independent of the RC criterion, but the accuracy of the resulting normal mode scattering parameters and corresponding eigenvalue solutions for a dispersion relation depends upon the ability of the selected mode set to satisfy the RC condition. Therefore, a technique to produce an eigenvalue equation of the form presented in (13) must be determined for the GSM to eliminate the numerical instabilities encountered with the GTM.

B. Dispersion Analysis Using the GSM

The procedure for determining the dispersion characteristics of a periodic circuit from a GSM is a new algorithm of significant importance, and it has been solved independently by Best [7] and Dai [18]. The eigenvalue problem defined by (13) for the GTM is similarly defined for the GSM. This is accomplished by defining a reference plane for the periodic circuit. Fundamentally, the circuit periodicity requires that the regions defining the input and output ports for the circuit must be identical with regard to waveguide type, waveguide cross-sectional dimensions, number of modes, and mode types and mode indices used for both regions. These restrictions were similarly imposed for the GTM dispersion analysis, and will make it possible to apply Floquet's theorem to the two port structure yielding

$$\begin{bmatrix} \overline{\overline{S_{11}}} & \overline{\overline{S_{12}}} - \overline{\overline{e^{-\Gamma_n L}}} \\ \overline{\overline{S_{21}}} - \overline{\overline{e^{\Gamma_n L}}} & \overline{\overline{S_{22}}} \end{bmatrix} \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \end{bmatrix} = \begin{bmatrix} \vec{0} \\ \vec{0} \end{bmatrix}. \tag{14}$$

This equation defines $\overline{\overline{e^{-\Gamma_n L}}}$ and $\overline{\overline{e^{\Gamma_n L}}}$ as diagonal matrices defining the eigenvalues for the system normal modes.

Fundamentally, (14) is a unique form for solution of the system eigenvalues, and simple manipulation of the equation will put it into the form of a Generalized Eigenvalue [19] defined as

$$(\overline{\overline{A}} - \lambda\overline{\overline{B}})\bar{x} = \bar{0}. \tag{15}$$

The manipulation of (14) into the form of a generalized eigenvalue equation is illustrated in Appendix A, which yields

$$\begin{bmatrix} \overline{\overline{1}} - \overline{\overline{S_{22}}}\,\overline{\overline{S_{11}}} & -\overline{\overline{S_{22}}}\,\overline{\overline{S_{12}}} \\ \overline{\overline{S_{11}}} & \overline{\overline{S_{12}}} \end{bmatrix} \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \end{bmatrix} = \begin{bmatrix} \overline{\overline{S_{21}}}\,\overline{\overline{e^{-\Gamma_n L}}} & \overline{\overline{0}} \\ \overline{\overline{0}} & \overline{\overline{e^{-\Gamma_n L}}} \end{bmatrix} \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \end{bmatrix}. \tag{16}$$

This form of the determinantal equation found by Best [7] was similarly determined by Omar [20] in support of Dai [18].

The determinantal equation for the GSM is shown in (16), as compared to (13) for the GTM. The fundamental difference between the two solution techniques is the implied numerical stability of the GSM due to the form of the matrix, while the presence of evanescent modes may force the GTM to become numerically unstable. Certain classes of problems involving only propagating modes yield identical solutions [15]–[17] from the two techniques. In general, the form of the mode matching solution requires the use of evanescent modes to satisfy the RC criterion. Thus, the GSM system normal mode solution technique is preferable to the GTM technique due to the possibility of encountering numerical instabilities with the latter.

IV. Dispersion Analysis of the Linear Vane-Type Waveguide Using the GSM

The dispersion properties of the linear vane-type waveguide have been previously explored using field matching algorithms [2], [4], [7], [8]. The linear vane-type waveguide geometry in this study is the closed end space geometry depicted in Fig. 1. The field matching algorithm developed by Gunderson [4] found resonances for the surface waves [6], [7] forming the first passband of the slow wave circuit. The ability to resolve the system normal modes and accurately solve for the dispersion of a periodic circuit using the GSM algorithm is determined by the number of modes kept in the truncation of the infinite series expansion of the waveguide junction normal modes.

The truncation of the infinite series expansion of the waveguide junction normal modes involves the ability of the modes used in the GSM simulation to satisfy the RC criterion. It has been demonstrated that if an insufficient normal mode expansion is used for a waveguide junction, then the calculated GSM will be in error due to the violation of the problem boundary and edge conditions [7], [11], [21]. Therefore, the ability of the GSM algorithm to adequately predict the dispersion characteristics of a periodic circuit is determined by the proper selection of region normal modes in the correct ratios between regions to satisfy the RC criterion.

The RC criterion problem is demonstrated with the mode matching simulation of a single period of the closed end space linear vane-type waveguide shown in Fig. 3. This geometry was also analyzed using the field matching algorithm developed by Gunderson [4] for the first passband. This circuit geometry has been extensively analyzed [7] using various experimental and analytical techniques. The intent of the present study is to provide a method to compare the field matching and GSM dispersion algorithms with experimentally determined dispersion information for the analysis of the structure shown in Fig. 3.

This problem was analyzed with the mode matching algorithm using three different mode sets to determine the dispersion properties as a function of the RC criterion. The modes used for these simulations consists of the rectangular waveguide TE^z and TM^z normal modes. The mode set selected for this mode matching simulation was based on the concept of the continuity of the transverse electromagnetic fields defined by an infinite series expansion of the waveguide region normal modes at the plane of a waveguide discontinuity. The infinite series expansion of the region normal modes requires truncation for implementation on a computer, in which the point of truncation is selected to satisfy the RC criterion. This concept is typically satisfied with the mode ratio algorithm [7], [11] to establish mode limits for the simulation. The mode ratio algorithm developed by Shih and Gray [22] was used to determine the mode index limits for this GSM simulation.

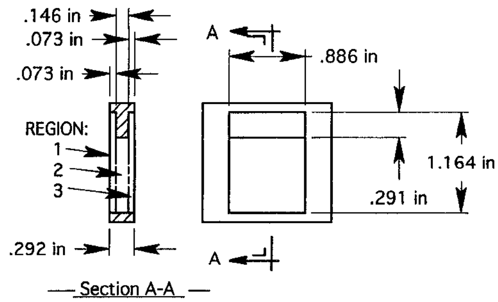

Fig. 3. Single period of linear vane-type waveguide with closed end spaces.

TABLE I
COMPARISON OF CALCULATED AND MEASURED DISPERSION PARAMETERS FOR THE LINEAR VANE-TYPE WAVEGUIDE SHOWN IN FIG 3

mode identifier	Experimental frequency, GHz	Gunderson [4] frequency, GHz	GSM 24 modes, GHz	GSM 52 modes, GHz	GSM 97 modes, GHz
$TE_{1,0}$	6 6600	not calculated	6 6600	6 6600	6.6600
$TE_{1,1}$ / $TM_{1,1}$	9 4800	not calculated	9 4800	9 4800	9 4800
$EH_{1\,1}$	8 6550	not calculated	8 5650	8 6100	8 6100
$EH_{1,2}$	11 6850	not calculated	11.9750	11.8050	11 8050
0-mode	6.6600	6 6682	6 6600	6 6600	6 6600
$\pi/9$-mode	6.9900	6.9753	6.9450	6.9750	6.9750
$2\pi/9$-mode	7 7700	7 7787	7 7250	7 7700	7 7700
$\pi/3$-mode	8 6550	8 6862	8 5500	8 6700	8.6700
$4\pi/9$-mode	9 4050	9 3234	9 3000	9 4200	9 4200
$5\pi/9$-mode	9 8100	9 7070	9 6900	9 8250	9 8250
$2\pi/3$-mode	10.0350	9.9435	9.9150	10 0500	10 0500
$7\pi/9$-mode	10 1850	10 0877	10 0650	10 2000	10 2000
$8\pi/9$-mode	10 2750	10 1662	10 1550	10.2900	10 2900
π-mode	10.2900	10.1911	10 1700	10.3050	10 3050

Three mode matching simulations were performed for this circuit, with the results shown for comparison in Table I. The GSM dispersion analysis made use of 24, 52, and 97 modes for Regions 1 and 3 shown in Fig. 3. The GSM dispersion algorithm was used to determine the cutoff frequency of the hybrid modes for the circuit, as well as slow wave mode dispersion information for the circuit shown in Fig. 3 assuming it were nine periods in length. This information is shown in Table I in comparison to experimental data as well as eigenvalue information using the field matching algorithm developed by Gunderson [4]. The error encountered using the field matching dispersion algorithm and the GSM dispersion algorithm with relation to the measured dispersion information is summarized in Table II.

The GSM simulations and circuit measurements were performed over a 6.0 GHz bandwidth extending from 6.0 GHz to 12.0 GHz using 401 frequencies. This provides a 15 MHz frequency of resolution for identifying mode frequencies. The experimental data presented in Table I serves as the point of reference for this comparison of dispersion analysis techniques.

The first GSM simulation involved 24 modes in Regions 1 and 3, and used 17 modes in Region 2. The Region 1 and 3 mode indices for the TE^z models included the $m_1 =$

TABLE II
COMPARISON OF THE DISPERSION ERROR OBTAINED BETWEEN MEASURED AND CALCULATED DATA SHOWN IN TABLE I FOR THE LINEAR VANE-TYPE WAVEGUIDE SHOWN IN FIG. 3

mode identifier	Gunderson [4] error, %	GSM 24 modes, error, %	GSM 52 or 97 modes, error, %
$TE_{1,0}$	N/A	0.0	0.0
$TE_{1,1}$ / $TM_{1,1}$	N/A	0.0	0.0
$EH_{1,1}$	N/A	1.0	0.5
$EH_{1,2}$	N/A	7.3	1.0
0-mode	0.1	0.0	0.0
π/9-mode	0.2	0.6	0.2
2π/9-mode	0.1	0.5	0.0
π/3-mode	0.3	1.2	0.2
4π/9-mode	0.8	1.1	0.2
5π/9-mode	1.0	1.2	0.1
2π/3-mode	0.9	1.2	0.1
7π/9-mode	0.9	1.2	0.1
8π/9-mode	1.0	1.2	0.1
π-mode	0.9	1.2	0.1

$m_3 = 0, 1, 2, 3$ and $n_1 = n_3 = 0, 1, 2, 3$ modes. Similarly, the TM^z mode indices for Regions 1 and 3 included the $m_1 = m_3 = 1, 2, 3$ and $n_1 = n_3 = 1, 2, 3$ modes. Region 2 for the TE^z modes was limited to mode indices $m_2 = 0, 1, 2, 3$ and $n_2 = 0, 1, 2,$ and for the TM^z modes to index limits $m_2 = 1, 2, 3$ and $n_2 = 1, 2$. Examination of the periodic waveguide system normal mode cutoff and slow wave mode dispersion information shown in Tables I and II indicates that the error using the GSM dispersion algorithm is greater than that obtained using the field matching dispersion algorithm.

The error in determining the dispersion information with the GSM algorithm can be reduced by increasing the number of modes used in the simulation to improve the RC criterion. The second GSM simulation involves 52 modes in Regions 1 and 3, and 38 modes in Region 2. The Region 1 and 3 mode indices for the TE^z modes included the $m_1 = m_3 = 0, 1, 2, 3$ and $n_1 = n_3 = 0, 1, 2, 3, 4, 5, 6, 7$ modes. Similarly, the TM^z mode indices for Regions 1 and 3 included the $m_1 = m_3 = 1, 2, 3$ and $n_1 = n_3 = 1, 2, 3, 4, 5, 6, 7$ modes. Region 2 for the TE^z modes was limited to mode indices $m_2 = 0, 1, 2, 3$ and $n_2 = 0, 1, 2, 3, 4, 5$ and for the TM^z modes to index limits $m_2 = 1, 2, 3$ and $n_2 = 1, 2, 3, 4, 5$. Examination of the dispersion information in Tables I and II indicates that the slow wave mode dispersion error has been reduced substantially by improving the RC criterion in this simulation.

Finally, the GSM dispersion error may potentially be further reduced by increasing the number of modes used for the simulation. The third GSM simulation involves 97 modes in Regions 1 and 3, and 71 modes in Region 2. The Region 1 and 3 mode indices for the TE^z modes included the $m_1 = m_3 = 0, 1, 2, 3, 4, 5, 6$ and $n_1 = n_3 = 0, 1, 2, 3, 4, 5, 6, 7$ modes. Similarly, the TM^z mode indices for Regions 1 and 3 included the $m_1 = m_3 = 1, 2, 3, 4, 5 6$ and $n_1 = n_3 = 1, 2, 3, 4, 5, 6, 7$ modes. Region 2 for the TE^z modes was limited to mode indices $m_2 = 0, 1, 2, 3, 4, 5, 6$ and $n_2 = 0, 1, 2, 3, 4, 5$, and for the TM^z modes to index limits $m_2 = 1, 2, 3, 4, 5, 6$ and $n_2 = 1, 2, 3, 4, 5$. Examination of the dispersion data presented in Tables I and II reveals that the dispersion solution converged for the frequency span being modeled using 52 modes.

Therefore, the system normal mode eigenvalue and dispersion solution determined using the mode matching algorithm forming a GSM has illustrated convergence using 52 modes, and the solution accuracy does not improve using 97 modes. This clearly illustrates the importance of the RC criterion for accurately solving for the surface wave mode eigenvalues and corresponding dispersion information from the GSM. The resulting determinant of the GSM using the PEC boundary condition with either 52 or 97 modes is shown in Fig. 4, and the corresponding system normal mode dispersion information is shown in Fig. 5. Fig. 4 clearly identifies the resonance for the 0-mode at 6.6600 GHz, and for the π-mode at 10.305 GHz. These resonances correspond to the beginning and ending frequencies associated with the first slow wave mode passband of the periodic linear vane-type waveguide circuit shown in Fig. 3. The dispersion of the hybrid waveguide modes shown in Fig. 5, in contrast to the dispersion of the system normal mode (slow wave circuit mode), illustrates the generality of this new analytic technique for periodic circuits. These hybrid waveguide modes have been previously addressed with regard to achieving low attenuation [5] in high power guided wave structures. The GSM dispersion and eigenvalue algorithms yield analytic expressions for the periodic circuits being analyzed, and the accuracy of the solutions are a function of the frequency resolution of the simulation, and the RC criterion.

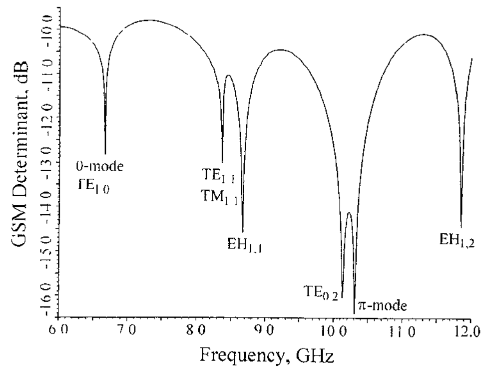

Fig. 4. Eigenvalues determined using the PEC boundary condition with the GSM for the linear vane-type waveguide shown in Fig. 3.

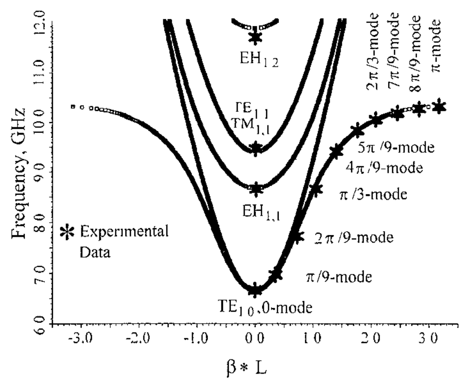

Fig. 5. GSM dispersion diagram determined for the first passband of the linear vane-type waveguide shown in Fig. 3 using either 52 or 97 modes for Region 1 of the circuit.

Fundamentally, the variational field matching solution approach developed by Gunderson [4] and recently determined by McVey [8] provides the fastest solution technique for solving for the surface wave dispersion properties of the linear vane-type waveguide. The solution of the π-mode typically suffers a 1% error between theory and experiment. This design error is acceptable for many applications. The cost associated with iteratively modifying the hardware to reduce this design error is typically prohibitive when higher accuracy dispersion characteristics are required. This dispersion design problem is easily resolved by completing the design process with the GSM dispersion algorithm, which will serve to identify competing modes for the surface wave mode passband demonstrated in Fig. 5.

Inspection of the dispersion diagram in Fig. 5 reveals that three waveguide modes exist in the passband for the surface wave mode. These modes ($TE_{1,0}$, $EH_{1,1}$ and $TE_{1,1}/TM_{1,1}$) are capable of interfering with the excitation and propagation of the surface wave on the circuit. These waveguide modes can be excited by circuit perturbations and coupling port geometries [7] which will lead to interference and power loss from the surface wave. The power loss from the surface wave is associated with coupling to the waveguide modes. The new dispersion analysis tool can be used to optimize the design of a periodic circuit, such as the vane-type waveguide, to eliminate the possibility of waveguide modes existing in the passband of the surface wave.

V. Summary

A generalized analytic technique has been developed using the mode matching algorithm to model a single period of a periodic circuit. This analysis results in the formation of a GSM for the circuit being analyzed. The resulting GSM is easily manipulated to produce an eigenvalue solution and a new dispersion solution for the system normal modes for the periodic circuit under study. The accuracy of these analytic techniques is determined by the RC criterion and the frequency resolution of the simulation. However, this analytic technique can easily be applied to any periodic circuit configuration capable of being analyzed with the mode matching algorithm, providing an analytic expression for the dispersion and eigenvalues for an arbitrary circuit configuration. The application of this concept to a linear vane-type waveguide yielded a surface wave dispersion error range of 0.0% to 0.21%. This analytic technique is easily implemented using conventional computing capability to provide a highly accurate eigenvalue and dispersion solution technique for designing new periodic circuits.

Appendix A
Derivation of the Determinantal Equation for the GSM

Equation (15) for the generalized eigenvalue equation [19] illustrates that the eigenvalues (λ) are determined for a system of equations split into two arguments, matrices $\overline{\overline{A}}$ and $\overline{\overline{B}}$. Restating (14) as two equations yields

$$\Rightarrow \overline{\overline{S_{11}}}\ \vec{a}_1 + \overline{\overline{S_{12}}}\ \vec{a}_2 = \overline{\overline{0}}\ \vec{a}_1 + \overline{\overline{e^{-\Gamma_n L}}}\ \vec{a}_2 \quad \text{(A1)}$$

and

$$\vec{a}_1 + \overline{\overline{0}}\ \vec{a}_2 = \overline{\overline{S_{21}}}\ \overline{\overline{e^{-\Gamma_n L}}}\ \vec{a}_1 + \overline{\overline{S_{22}}}\ \overline{\overline{e^{-\Gamma_n L}}}\ \vec{a}_2. \quad \text{(A2)}$$

Equations (A1) and (A2) can be stated in matrix form as

$$\begin{bmatrix} \overline{\overline{S_{11}}} & \overline{\overline{S_{12}}} \\ \overline{\overline{1}} & \overline{\overline{0}} \end{bmatrix} \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \end{bmatrix} = \begin{bmatrix} \overline{\overline{0}} & \overline{\overline{e^{-\Gamma_n L}}} \\ \overline{\overline{S_{21} e^{-\Gamma_n L}}} & \overline{\overline{S_{22} e^{-\Gamma_n L}}} \end{bmatrix} \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \end{bmatrix}. \quad \text{(A3)}$$

The resulting equation is generally of the form defined in (15). The problem is further simplified for solution if (A1) is rewritten as

$$\overline{\overline{S_{22}}}\ \overline{\overline{S_{11}}}\vec{a}_1 + \overline{\overline{S_{22}}}\ \overline{\overline{S_{12}}}\vec{a}_2 = \overline{\overline{0}}\vec{a}_1 + \overline{\overline{S_{22}}}\ \overline{\overline{e^{-\Gamma_n L}}}\vec{a}_2. \quad \text{(A4)}$$

Subtraction of (A4) from (A2) yields

$$\left(\overline{\overline{1}} - \overline{\overline{S_{22}}}\ \overline{\overline{S_{11}}}\right)\vec{a}_1 - \overline{\overline{S_{22}}}\ \overline{\overline{S_{12}}}\vec{a}_2 = \overline{\overline{S_{21}}}\ \overline{\overline{e^{-\Gamma_n L}}}\vec{a}_1. \quad \text{(A5)}$$

Equations (A1) and (A5) can be stated in the form of the Generalized Eigenvalue problem as

$$\begin{bmatrix} \overline{\overline{1}} - \overline{\overline{S_{22}}}\ \overline{\overline{S_{11}}} & -\overline{\overline{S_{22}}}\ \overline{\overline{S_{12}}} \\ \overline{\overline{S_{11}}} & \overline{\overline{S_{12}}} \end{bmatrix} \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \end{bmatrix} = \begin{bmatrix} \overline{\overline{S_{21}}}\ \overline{\overline{e^{-\Gamma_n L}}} & \overline{\overline{0}} \\ \overline{\overline{0}} & \overline{\overline{e^{-\Gamma_n L}}} \end{bmatrix} \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \end{bmatrix}. \quad \text{(A6)}$$

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